TEACHING THEORY BUILDING[1]

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The Importance of Dynamic Modeling

The basis of much of the work at TERC (the Technical Education Research Centers) is that kids learn by doing science, not by learning science facts and problem types, but by participating in a meaningful way as apprentice scientists and mathematicians. The more we can bring aspects of scientific and mathematics research and original work into the classroom, the more kids will like science and learn scientific skills, ideas, and approaches.

Creating theories and testing them against reality are the two major activities of science and mathematics. If we are to give students a realistic view of science, we should find ways to foster both their theory-building and experimentation. While we have had real successes using technology in the form of microcomputer-based labs (MBL) to aid experimentation, one of the most challenging and least-explored educational goals is to help students learn theory-building and testing. An important long-term goal at TERC is to create environments where students can build and test their own theories. This is the work of several activities we call modeling projects.

We think students should learn the process of construction, testing and refining theories or models as part of their introduction to mathematics and science. This is motivating because it allows students to participate in an important part of mathematics and science and it is important socially because models are an increasingly important aspect of social policy determination. For instance, the entire

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debate about global climate change centers on the accuracy of predictions of various models. Even though global models are based on advanced mathematics and physics, many such models can be made broadly accessible because, with the help of microcomputers and appropriate material, they can be set up and solved without dealing with the formalism of calculus.

Making wider use of computer-based modeling could have a major impact on college and pre-college science, mathematics, and social science education. Interesting and complex topics could be introduced earlier in the science curriculum and mathematics could build on a base of numerical methods to teach formal calculus earlier and more effectively. The study of numerical solutions to differential equations is usually postponed until after students have a good grounding in calculus, but because of the availability of microcomputers, this sequence is not necessary. The order can be reversed; the kinds of thinking required in model building and the understanding of how a model changes over time can provide the conceptual framework for calculus. What is more important, student ability to solve dynamic systems can be used to enhance and restructure the pre-college curriculum in science, mathematics, social sciences and technology. An extremely broad range of interesting problems can be solved numerically: world models, ecological systems, chemical reactions, classical particle dynamics, business cycles, and much more. These are interesting issues that are seldom addressed in introductory courses in ways that allow students to understand the models, their utility and limitations.

Teaching students to create quantitative theories is clearly an ambitious goal that is both important and fraught with difficulties. Much of educational practice mitigates against student theory-building; it is a skill that is difficult to assess, it involves asking questions that do not have simple right answers, and it often requires extensive computational capacity not usually available or understood by teachers. Still, we feel that technology removes some of the barriers to formulating and evaluating theories and the potential gains that student mastery of theory-building would achieve make it important to learn more about how this topic could be brought into the classroom.

Our first formal effort to explore these ideas was the Modeling Project that was largely exploratory; we wanted to learn what is possible technically and pedagogically, and we wanted to evaluate what is possible in school settings. We used several different computer representations of dynamic systems, including spreadsheets, STELLA, and our own software, in conjunction with MBL. In so doing, we developed some very interesting software and curriculum materials that should be of value to teachers, curriculum planners and educational researchers.

Project Strategies

The Modeling Project first explored ways advanced computer technology could make dynamic modeling accessible to high school students and then worked on the curriculum and in-service training implications of this capability.

Our approach in teaching systems dynamics at the pre-college level is to attempt to lower the level of abstraction with which students must deal in solving models. This project represented a search for alternative representations of the ideas of rate and change that are more concrete and accessible and do not require formal analysis, support of the microcomputer:

- Avoiding Calculus Notation. Calculus notation can be avoided through the device of flow diagrams where flow in pipes represents derivatives (i.e., rates of change of variables) in a systematic way that does not require the formulas or nomenclature of calculus. Calculus can also be avoided by using difference methods.

- Avoiding Algebraic Notation. The algebra of systems of equations was represented through the topology of interconnections in flow diagrams. The actual functional details of these interactions, usually requiring a fair amount of algebra, were represented graphically wherever possible.

- Using Concrete Problems. We used microcomputer-based labs to generate data because we feel that it is particularly meaningful and accessible to students. MBL also can familiarize students with the graphical representation of data used to specify the functional relations in the model building phase.

- Using good software. We designed and prototyped *Models*, a fast, easy to use software tool for the solution of system dynamics problems. A variety of input and output formats make the tool potentially flexible and widely useful. The tool is designed to simplify the mechanics of problem solution and allow students to focus on model building.

We were particularly interested in the synergistic effect of system dynamics and MBL used together. Students learn (and science progresses) through the interplay of theory and experimentation. Systems dynamics provides a powerful tool for theory-building, while MBL provides vastly easier access to the phenomena of science. Together, they have the potential to give students a better understanding of more science that is relevant, interesting and motivating.

Representations of Dynamic Systems

There was considerable work in making dynamic modeling accessible to naive students prior to this project. Faculty of the Sloan School at MIT have a long history of teaching modeling ideas to business students (Goodman, 1974 and Edward Roberts, 1978). They have developed a language called Dynamo (Richardson, 1981) that simplifies the specification and analyses of models. Modeling project co-director

Nancy Roberts has used a version of Dynamo adapted to microcomputers called microDynamo and has developed teaching materials that allow students as early as ninth grade to set up and solve modeling problems with micro-Dynamo (Roberts et al, 1983).

Two techniques that facilitate mathematically naive students' ability to use models grew out of this experience of teaching modeling. One technique, illustrated in Figure 1, is to represent a system as a causal-loop diagram (Richardson, 1991). These diagrams help students focus on the relevant factors in a model, identify cause-and-effect relationships, and determine the sign of the feedback for each loop. Grade school children are able to translate systems into causal-loop diagrams and to predict the gross behavior of a system based on these diagrams. Causal loop diagrams are useful in identifying the relevant variables and the type of solution expected. However, they are hard to convert into quantitative models because they make no distinction between simple variables and integrals. As a result, we made little use of causal-loop diagrams in this project.

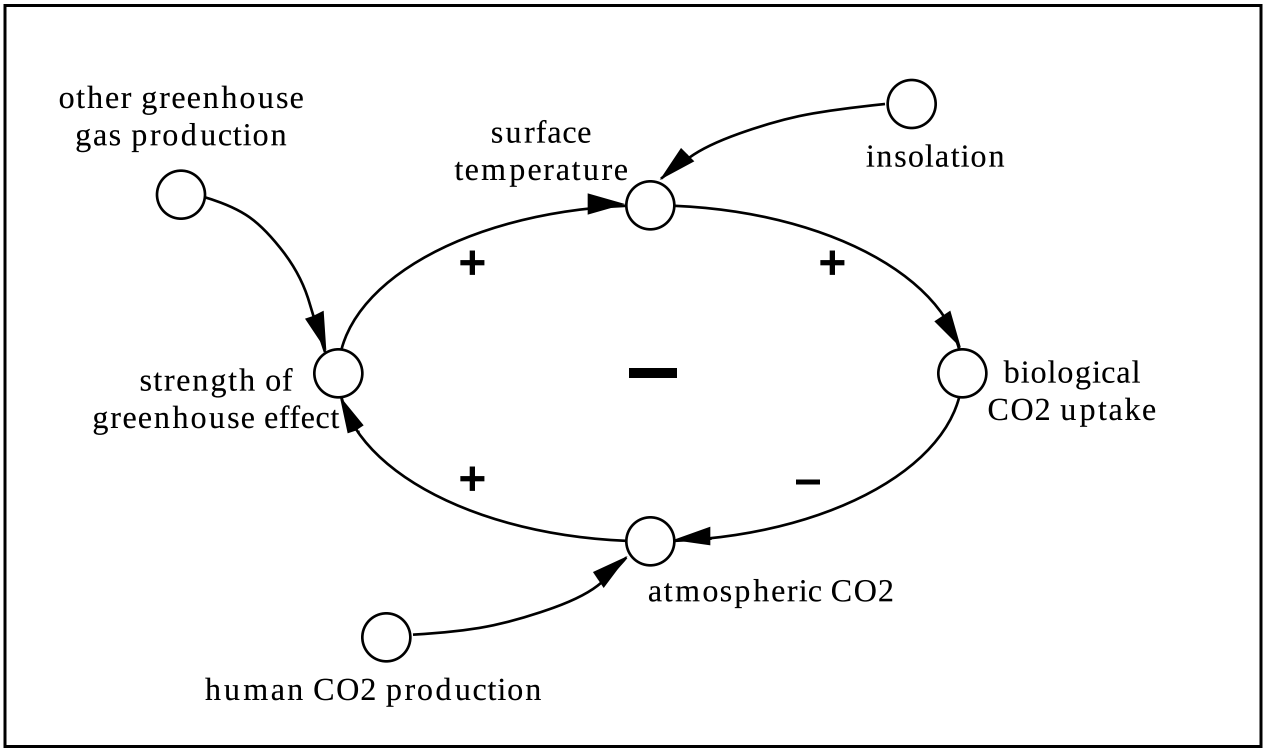

Figure 1: A simple model of the net energy flow into and out of the Earth represented in causal loop form. The solution is expected to be stable, as indicated by the negative in the center.

A second intermediate construct that has proven more useful is flow diagrams. This is an iconically-based representational system that allows students to analyze quantities and their derivatives without explicitly using calculus concepts by using symbols such as reservoirs and valves borrowed from chemical engineering. The example in Figure 2 shows the system in Figure 1 represented as a flow diagram.

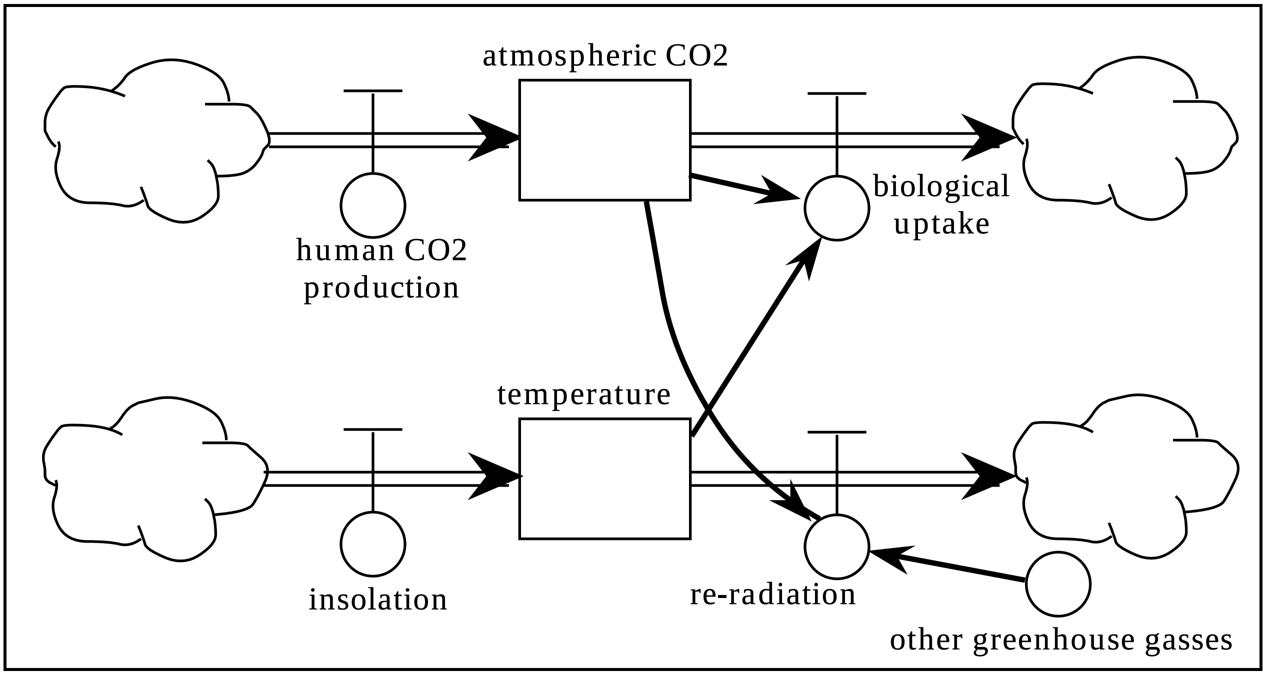

Figure 2: A flow diagram corresponding to Figure 1.

Flow diagrams express relationships that can be relatively easily translated into algebraic expressions because they represent the fundamental concept of integration and distinguish variables whose values are determined by integration from simple algebraic variables. As a result, flow diagrams are useful constructs that were used to generate inputs to modeling programs such as micro-Dynamo. For many problems this translation process requires the kind of algebra usually studied in a first-year algebra course. One of the innovations envisioned in this project was to automate the process of converting a flow diagram into input for the computer. While the National Science Foundation was considering the proposal that eventually led to this project, High Performance Systems, Inc. did just that with their program, STELLA. This proved to be a boon to the project, since STELLA was close enough to what we needed to permit us to begin testing with students from the beginning of the project.

One of the surprising results of the project is that students cannot grasp flow diagrams as easily grasped as we had hoped. The two types of variables–algebraic and integral–are not easily distinguished by naive students, even when they are given the more descriptive names, rates and levels. The importance of the pipes, valves and reservoirs is that the quantity flowing through the pipes, regulated by the valves and accumulating in the reservoirs is conserved. Unfortunately, this metaphor is probably meaningful only if students have experience with fluids in this kind of apparatus. There is nothing in the diagram, even in STELLA’s animation mode, that suggests something moves through the pipes and valves into and out of the reservoirs. Perhaps if more student attention was drawn to the details of valves, pumps and reservoirs, and time devoted to exploration with actual, physical operating versions of these, the iconic versions would be more meaningful.

Some of the crucial concepts of calculus are hidden in the valve. If there is water flowing through the pipe, then the valve controls the rate of flow. This control is not an altogether accurate model of real valves because the flow in the model is considered to be independent of pressure; in fact, pressure plays no role at all. As a result, two valves cannot be placed in series in the model; a restriction that is clearly non-physical. Thus, it is not clear whether a great deal of experience with physical systems would help or hinder student understanding of flow diagrams.

However, there are problems with flow diagrams when non-hydraulic systems are being considered. If the quantity "flowing" is new-born rats or the incident flux of radiation to the earth, the relation between rate and quantity can easily become confused. Quantities that require second derivatives, such as position in mechanics problems, are a problem. The valve-and-tank system represents only first derivatives, so two, coupled valves and tanks are required with the intermediate derivative–velocity in this example–appearing as a rate in one part of the diagram and a level in another as illustrated in Figure 3. Furthermore, velocity and acceleration are fairly abstract quantities to be flowing and accumulating. Worse, they can be negative, so negative quantities can accumulate and negative flows can be generated. Figure 3 would have to be more complex in STELLA because the same variable name cannot be used twice, even when identity is intended. While all this is mathematically correct, these problems limit the applicability of the stock and flow model for naive students; the representation is probably better as a reminder for students who have already mastered the formalism of calculus. Since few young students are familiar with these concepts, the symbols can be little more than mnemonics for some poorly defined ideas.

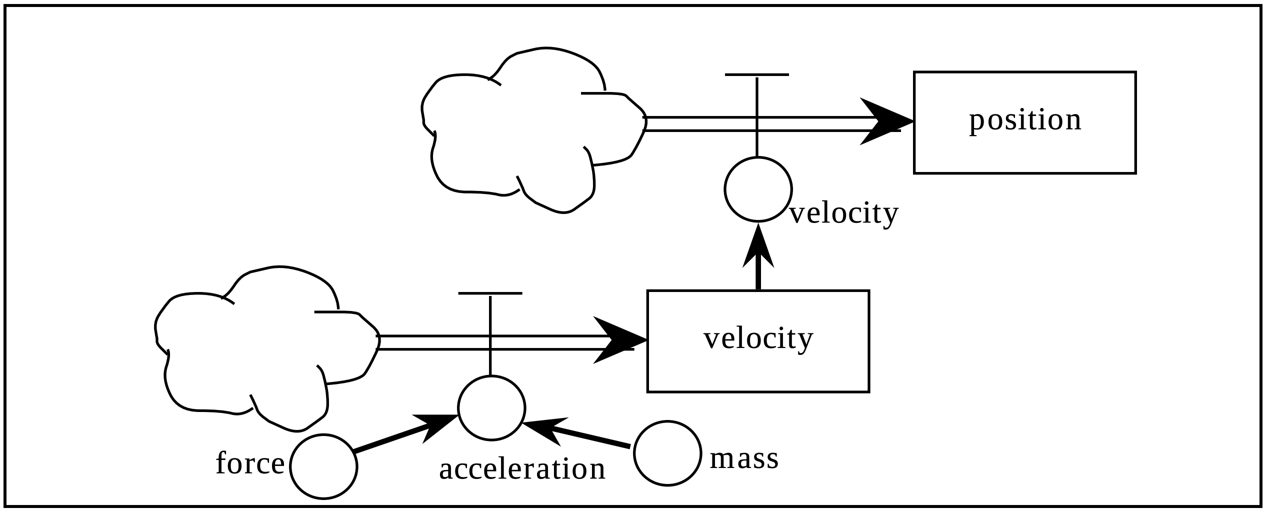

Figure 3: Newton's Second Law as a flow diagram showing velocity as both a rate and level quantity. Newton's Law expressed as a=F/m, is hidden inside the

acceleration circle. What does it mean to have velocity flow through a valve called acceleration controlled by force and mass?

Concrete Experiences Through MBL

For some time the staff at TERC has been exploring educational applications of computer-based, real-time data acquisition, an application TERC named microcomputer-based labs (MBL). This work was motivated by the dream of developing a series of low-cost probes that could be used by students to measure the widest possible range of variables: temperature, humidity, distance, velocity, acceleration, force, pressure, pH, light, air flow, rotation, radiation, etc. These should be able to be measured in time scales ranging from microseconds to years, singly and in arbitrary groupings. In the same ways that this instrumentation has increased the efficiency and scope of practicing scientists and engineers, it should also improve student learning in experimental settings, making learning more effective and providing a more accurate view of the conduct of science.

Perhaps an example can help illustrate the power of this approach. One of the sensors we developed is an ultrasonic motion detector using the electronics developed by Polaroid for their autofocus cameras. Jim Pengra, a physicist on leave at TERC from Whitman College, first connected the Polaroid sensor to a computer. He programmed the computer to tell the sensor to emit a “chirp” consisting of a few cycles of ultrasonic sound. This sound can reflect back to the sensor which detects the returning signal. Jim programmed the computer to measure the time between emitting and detecting the signal and, using the known speed of sound, to convert it to the distance between the two. By repeating this process up to 40 times per second, the computer can have detailed, accurate, and instantaneous data about the location of whatever is closest to the sensor. Jim programmed the computer to graph this distance as people, pendula, and carts moved around in front of the sensor. We quickly realized that we could compute and graph in real time velocity and acceleration from these data.

The resulting system is dramatic: a student can walk up to the sensor and see a graph of his or her motion while moving. Misconceptions about the graphs melt away. For instance, a graph with negative velocity is a tough problem for most students to interpret. But by watching the velocity-time graph while walking, students see very quickly that movement away from the sensor generates a positive velocity and movement towards it generates a similar negative one. While initially confusing, most students are quickly convinced of the logic of this, since the distance is getting smaller as you walk toward the sensor. In a few minutes, students at all grade levels quickly learn to interpret the graphs and relate them to their motion.

We were fairly certain that, by itself, the construction and solution of dynamic systems in STELLA would not be accessible to beginning students. Because the central idea of this approach rests on an intuitive understanding of the flow of an incompressible fluid through a valve and its accumulation in a tank, we suspected that some prior concrete experience with water, pipes and tanks might help make the STELLA models more salient. To address the need for concrete experience, we developed a set of experiments with "Leaky Buckets" and developed a low-cost microcomputer-based lab interface and software for the Macintosh for general data logging and display.

Leaky Buckets consisted of a system of beakers, graduated cylinders, tubes, tube clamps and colored water, together with an instrumented float, that could be used to set up simple dynamic systems and record the water level in one beaker on the computer. This system can be used in a number of ways to illustrate points about dynamic systems. One way of using the system is to advance time in discrete steps by having students perform some operation repeatedly and record the result. For instance, students could add to the beaker 10% of the volume of liquid in a beaker each time interval. If the cross-sectional area of the graduated cylinder is 10% of the area of the beaker, then all students have to do is bring the level in the cylinder to that of the beaker each iteration. This introduces iteration and time intervals and shows a simple system where the input rate is level-dependent, leading to an exponential. A second way to use the system is to allow the water to flow between two or more beakers, recording the water level in one. This could be interesting in itself or it might be model of some other system such as a stream with reservoirs or a manufacturing plant.

By using MBL to obtain real-time data that can be displayed side-by-side with the results of computational models, students could be both theoreticians and experimentalists, moving quickly between observation; and theory-building, and beginning to experience the full range of intellectual activities of practicing scientists. The real data are, hopefully, also motivational and a source of ideas for models and model details. Microcomputer-based labs also provide a degree of concreteness that help ground student model-building in reality and comprehensible physical actions.

Our initial strategy involved focusing student attention on the pipe reservoir relationship by examining actual pipes and reservoirs with Leaky Buckets. We devised a sequence that had students first measuring water levels and flows with pipes and burettes interfaced to the computer. A series of questions probed the performance of this system; in effect, the system was a model of itself. The next step was to use this water system to represent something else; initially, just a different system of water, such as large reservoirs, and then later, populations and bank balances. This introduced the idea of models, and specifically models based on rates and levels. It was then a relatively smaller step to graduate from the messy water system to the much cleaner and more easily manipulated cybernetic version.

We found that the combination of MBL and Leaky Buckets did not lead to a better understanding of stocks and flows. There may be some non-MBL activities involving working with liquids that are valuable and worth pursuing in support of student understanding of flow models. However, our later disenchantment with all stock and flow models led us to abandon this approach. On the other hand, the combination of MBL and modeling seems to be a rich one that needs further work. The commercial software packages were unable to combine experimental MBL data and theoretical results on the same graph. Our Models software was designed to overcome this shortcoming but was available too late in the project to study with students.

Spreadsheets and Calculations

The actual computations are hidden in STELLA, so that students and teachers who feel a need to understand in detail what mathematics is being used are left unsatisfied. There is good reason for hiding these details, since the software uses Runge-Kutta two and four-step algorithms that are far from transparent. However, the simple Euler approximation is not only accessible but almost self-evident. In order to make the computations clear to students, we developed a numerical approach using the graphing spreadsheet Excel.

The algebraic approach using Excel is so different from the stock and flow approach of STELLA that it was difficult for students to appreciate the ways in which the two were related. Furthermore, it was very hard for students to accurately set up stock and flow models in STELLA. The difference between a simple variable and a flow is difficult to grasp and completely hidden in the causal loop diagrams. Students regularly confused the two and therefore made little progress in creating a realistic STELLA model.

We are convinced by this experience that it is better to use an algebraic, spreadsheet-based approach for teaching dynamic modeling for a number of reasons. There seems to be a particular transparency to the operation of a spreadsheet that appeals to students. Its cellular nature makes all the calculations clear and accessible and its continuous updating gives extensive feedback to students. In addition, spreadsheets are increasingly used in education so educators are gaining familiarity with their operation. Time investments teachers make in learning one spreadsheet can be utilized in many places in the curriculum and can be transferred to new and more powerful software and hardware as it becomes available.

An Integrated Software Package

The use of all three different software environments (MBL, STELLA and Excel) tended to confuse students as much as it clarified systems ideas. Therefore, we devoted considerable resources to the definition and piloting of an integrated software environment called *Models* that combined the best of all three. Unfortunately, this was a difficult software task undertaken at a time when the software development tools for the Macintosh were relatively primitive. As a result, it was not possible to complete all aspects of *Models* and the software was not available in time for integration into the materials and teacher training efforts. The *Models* design represents an important area for future development.

Our educational strategy, then, was to move from the concrete to the abstract, and from simple models to more complex ones. This general strategy can be applied to mathematics and all the sciences. Since there is no room in the curriculum for teaching modeling tools by themselves, we decided to develop curriculum materials that use these general strategies within all the different disciplines. This variety of material both illustrates the power of the approach and also provides a practical means for faculty in any discipline to incorporate the material into their curriculum.

Moving into the Classroom

Our first question was whether typical students could, by early high school, describe complex dynamic systems graphically and then use the computer to solve the resulting systems. We found that students as early as ninth grade could do this, building dynamic models and understanding their solution, in effect using the concepts of calculus without knowing its formalism. We used several different computer representations of dynamic systems, including spreadsheets and programs that incorporated the flow diagram approach developed by the System Dynamics group at MIT. It appeared that spreadsheets gave the most accessible representation of dynamic systems, and that the “flow” representation had weaknesses and needed additional research.

Our second question was how this system modeling approach could be used in instruction. We reasoned that it would be difficult for isolated teachers to begin using modeling because the curriculum time investment it entails might require too much modification of any one course. We also felt that modeling concepts would be easier for a group of teachers in one school to introduce together. Thus, we developed curriculum materials for use in mathematics, physics, biology and social studies and a simple introduction to modeling that a teacher in any of these courses could use.

Feeling that we had discovered something important to tell teachers, we made a major effort to disseminate our approach to modeling. Because the project involved math, science, and technological concepts that are not widely known in the teaching profession, we faced a difficult communications problem. In order to help explain our perspective, we prepared a 20-minute videotape *"How it Ought to Be”* showing kids working with our materials[1]. We also devoted substantial sections of our newsletter, *Hands On!,* to the project, and gave numerous workshops, lectures, and papers on project-related activities.

We experimented directly with teacher development and school implementation of various approaches to dynamic modeling. In order to assess training and

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[1] his tape is available from Mass Educational Television, 75 Acton Street Arlington, MA 02174.

implementation issues surrounding modeling, we worked with faculty from several schools. In one case, we introduced the material we developed to teachers from two schools in a four-day summer workshop and followed this up with six weekly in-service sessions. However, few of these teachers integrated the material in their teaching; it appears we underestimated the in-service time required to acquire an appreciation for the mathematics, the computer skills required and the difficulty in instituting the required curriculum changes.

It is now clear that the institutional and teacher support issues represent the major barriers to wide use of dynamic modeling and to the potential restructuring of the mathematics and science curriculum this could enable. This project has shown that students completing a first course in algebra have the conceptual tools required to use available technology to understand dynamic modeling. There are some barriers to wider use of this approach that are technical, involving computer interface issues that need investigation. The solution of these technical problems would make modeling easier to disseminate. However, the main barrier to wider use of dynamic modeling is that its effective use in the curriculum requires teachers to make major conceptual and mathematical changes that take time and resources. Here is yet another situation where we know how major improvements could be made in education but we currently lack the resources and the human talent needed to widely implement these improvements.

Future Directions

Developing strategies for infusing modeling throughout the curriculum is a long-term commitment of TERC's. We will continue to work on the material and look for ways to incorporate modeling approaches into teacher training. We have underway a research project, Measuring and Modeling, which will contribute to our understanding of how students might learn calculus concepts, creating models for situations that are accessible for experimentation and measurement.

The next major step in developing and disseminating this approach to dynamic modeling needs to be a long-term demonstration project. Best done in association with a group of schools that want to restructure their math and science curriculum along the lines permitted by a thorough integration of computer-based modeling, such a project would need ample resources for teacher training and administration orientation, for the development of new assessment tools, and for time to allow the schools to make a series of curriculum changes that would accommodate the changing capacities of their students and faculty.

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